Design of multichannel DWDM fiber Bragg grating filters by Lagrange multiplier constrained optimization

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Abstract: We present the synthesis of multi-channel fiber Bragg grating (MCFBG) filters for dense wavelength-division-multiplexing (DWDM) application by using a simple optimization approach based on a Lagrange multiplier optimization (LMO) method. We demonstrate for the first time that the LMO method can be used to constrain various parameters of the designed MCFBG filters for practical application demands and fabrication requirements. The designed filters have a number of merits, i.e., flat-top and low dispersion spectral response as well as single stage. Above all, the maximum amplitude of the index modulation profiles of the designed MCFBGs can be substantially reduced under the applied constrained condition. The simulation results demonstrate that the LMO algorithm can provide a potential alternative for complex fiber grating filter design problems.

OCIS codes: (000.3860) Mathematical methods in physics; (060.2340) Fiber optics components; (230.1480) Bragg reflectors;

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1. Introduction

Fiber Bragg gratings (FBGs) are essential optical devices both for fiber communications and sensor applications due to their powerful ability to act as narrowband filters, optical add-drop multiplexers, dispersion compensators and cavity mirrors in fiber lasers [1]. Among these, superstructure or sampled FBG filters are especially attractive for dense wavelength-division-multiplexing (DWDM) applications in the existing long-haul fiber network due to their comb filter response [2], [3].

With a periodic sampling function for the single channel seed grating profile, one can generate a multiple channel reflection spectrum by a sampled FBG. Subsequently, it has been found that sampled FBGs could be extended to compensate simultaneously for both the dispersion slope and the dispersion itself [4-6]. However, in comparison to a single channel grating, manufacture of an N-channel FBG device requires larger variation of the photo-induced refractive index change. In fact, it has been shown that the total index change is directly proportional to the number of the constituent gratings to be written, N times higher than a single channel grating [2]. Since the maximum index change with UV irradiation in silica glass is in the order of 0.001, there is an upper bound on the number of gratings that can be written by the superposition method in practical fabrication. Therefore, phase sampling is a preferred method for high-channel count FBG designs as the index modulation requirement for optimized gratings is only a square root growth with the number of channels, $\sqrt{N}$ times higher [7], [8]. In the literature, there have been many developed optimization-based or inverse design methods for fiber Bragg grating filters [9-12]. Among these methods, the inverse scattering discrete layer-peeling (DLP) algorithm has been used to directly design multichannel fiber gratings with an additional simulated annealing optimization process for different channel phases [13]. Recently, a general design method based on a genetic algorithm has also been applied to design the multichannel optical add-drop multiplexer as well as its dispersion shifts [14].

However, for these approaches, the simplicity, efficiency, and direct synthesis advantages of the DLP algorithm are faded due to the inclusion of an additional Monte-Carlo approach (simulated annealing or genetic algorithms). What is more, all these methods mentioned above are complicated and lacking the design flexibility for practical applications. In other words, the flexibility of spectrum-tailoring for filter design, fabrication and packaging is limited.

In this study, a new optimization-based approach for synthesizing the MCFBG filters for DWDM applications is investigated. The approach is based on a simple algorithm, the

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Lagrange multiplier optimization (LMO) method, which can constrain various parameters of the designed devices for the practical application demands through a user-defined cost functional. In general, when compared to the layer-peeling algorithm, our proposed method can easily embed various constrains in the cost functional. When compared to Mote-Carlo based approaches such as the genetic algorithms or others, our method is a direct synthesis method without using random numbers and thus a smoother coupling coefficient profile as well as faster convergence speed can be obtained. Moreover, by varying the weighting parameters in the user-defined cost functional, the index modulation requirements can be controlled to meet real fabrication conditions. The LMO method has been proved to be very useful in designing optical pulse shapes to achieve various goals [15-17]. The main aim of the present work is to carry out the theoretical framework and to demonstrate the suitability and advantages of this method for advanced FBG filter design problems. In the study, flat-top MCFBGs with arbitrary channel spacing and low dispersion for DWDM applications can be obtained with the additional advantage of reducing the index modulation for commercially available photosensitivity fibers. The convergence rate is very fast and direct for the LMO algorithms [18-19] when compared to the stochastic approaches. The design examples in this work prove that the LMO algorithm is an effective method for optimally designing complicated fiber grating devices.

2. LMO algorithm for FBG design

Our LMO method is based on the conventional coupled-mode equations for FBGs [1]

$$\frac{dR(\delta,z)}{dz} = i\delta \cdot R(\delta,z) + i\kappa(z)S(\delta,z) \tag{1a}$$

$$\frac{dS(\delta,z)}{dz} = -i\delta \cdot S(\delta,z) - i\kappa^*(z)R(\delta,z) \tag{1b}$$

where the amplitudes R and S are the forward- and backward mode amplitudes , $\delta = \pi\left(\frac{2n_0}{\lambda} - \frac{1}{\Lambda}\right)$ is detuning, λ is wavelength and Λ is the grating period. The parameter κ(z) = $\eta\pi\Delta n(z)/\lambda_c$ is the designed coupling coefficient distribution function with Δn(z) being the envelope function of the grating index modulation, λ_c is the central wavelength and η the overlapping factor. In this study the κ(z) function will be assumed to be real for the ease of practical fabrication. The main idea of the LMO method is to define an objective functional that needs to be minimized, such as,

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$$
\begin{aligned}
J &= \frac{1}{2}\int_{-\infty}^{\infty} \left[r(\lambda) - r_d(\lambda)\right]^2 d\lambda + \frac{\beta}{2}\int_0^L \left[\kappa(z)\right]^2 dz \\
&+ \int_0^L \int_{-\infty}^{\infty} \mu_{R,R} \cdot \mathrm{Re}\left[\frac{dR}{dz} - i\delta R - i\kappa S\right] d\lambda dz \\
&+ \int_0^L \int_{-\infty}^{\infty} \mu_{R,I} \cdot \mathrm{Im}\left[\frac{dR}{dz} - i\delta R - i\kappa S\right] d\lambda dz \\
&+ \int_0^L \int_{-\infty}^{\infty} \mu_{S,R} \cdot \mathrm{Re}\left[\frac{dS}{dz} + i\delta S + i\kappa^* R\right] d\lambda dz \\
&+ \int_0^L \int_{-\infty}^{\infty} \mu_{S,I} \cdot \mathrm{Im}\left[\frac{dS}{dz} + i\delta S + i\kappa^* R\right] d\lambda dz
\end{aligned} \tag{2}
$$

Where $r(\lambda) = |S(0)/R(0)|^2$ is the calculated reflection spectrum, L is the total length of the grating, and β is a positive number acting as a weighting parameter for the constraint control. In the defined cost functional, Eq. (2), the spatially coupling coefficient κ(z) is used to shape an output reflection spectrum $r(\lambda)$ of a given reflection spectrum $r_d(\lambda)$ and to minimize both the reflection spectra difference and the norm of the coupling coefficient profiles simultaneously.

In the proposed method, the forward / backward-modes and the Lagrange multipliers are separated into real and imaginary parts, respectively, i.e. $R = R_R + iR_I$, $S = S_R + iS_I$, $\mu_R = \mu_{R,R} + i\mu_{R,I}$ and $\mu_S = \mu_{S,R} + i\mu_{S,I}$.

To minimize the cost functional J , a variational method for Eq (2) is used with respect to the forward- and backward- modes R and S through the Lagrange multipliers μ_R and μ_S . The resulting equations of motion for the Lagrange multipliers are

$$
\frac{\partial \mu_R}{\partial z} = i\delta \cdot \mu_R - i\kappa\mu_S \tag{3a}
$$

$$
\frac{\partial \mu_S}{\partial z} = -i\delta \cdot \mu_S + i\kappa^* \mu_R \tag{3b}
$$

with the boundary conditions obtained by varying R and S at z=0,

$$
\mu_R(0) = -R(0)\frac{2r\Delta_r}{R_R^2 + R_I^2} \tag{4a}
$$

$$
\mu_S(0) = S(0)\frac{2\Delta_r}{R_R^2 + R_I^2} \tag{4b}
$$

where $\Delta_r = r(\lambda) - r_d(\lambda)$ is the discrepancy between the output and target reflection spectrum. Then, the cost functional J is varied with respect to the coupling coefficient function κ(z)

$$
\frac{\delta J}{\delta \kappa^*} = \beta \cdot \kappa + i\int_{-\infty}^{\infty} \left(\mu_R S^* - \mu_S^* R\right) d\lambda \tag{5}
$$

Finally, the Eq. (1)-Eq. (5) are solved in a self-consistent way with the following procedures:

(a) Guess an initial $\kappa_{ini}(z)$ and let $\kappa_{old}(z) = \kappa_{ini}(z)$.
(b) Solve the Eq. (1) and R(z) and S(z) can be obtained from $z = L$ to $z = 0$.

(c) Set the boundary conditions of $\mu_R(0)$ and $\mu_S(0)$ by using Eq. (4). Then, the propagations of the Lagrange-multiplier functions $\mu_R(z)$ and $\mu_S(z)$ from $z=0$ to $z=L$ can be obtained by solving the Eq. (3).

(d) Find $\delta J/\delta\kappa^*$ from Eq. (5) and update the new medium

$$\kappa_{new}(z)=\kappa_{old}(z)-\alpha\frac{\delta J}{\delta\kappa^*} \tag{6}$$

where α is an ad hoc constant.

(e) Repeat the steps (b) to (d) until convergence are reached.

3. Design results and discussion

In order to evaluate the effectiveness of the proposed LMO algorithm for FBG filter design, several MCFBG filters with different grating length, channel spacing and bandwidth are presented in this section. All of the MCFBG filters are designed with the LMO algorithm described in the previous section with a same initial Gaussian apodization profile for the coupling coefficient. In the designed MCFBGs, the target spectrum of the reflectivity is set to be

$$r=\sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} r_0\cdot\exp\left\{-\left[\frac{\lambda-\left(\lambda_c+(\frac{2m+1}{2})\cdot\Delta_{CS}\right)}{\Delta\lambda}\right]\right\}^{20} \tag{7}$$

where N is the total number of the channels, r_0 is the maximum reflectivity, λ_c is the central wavelength, Δ_{CS} is channel spacing, and $\Delta\lambda$ is the bandwidth for each channel. The total number of the calculated spectral points is set to be 200 and the central wavelength is set to be 1.55×10^{-3} mm (1550nm). The units of λ and L are mm, and κ(z) is mm^{-1}. In the LMO algorithm for synthesizing MCFBGs, α is an ad hoc constant and β is a weighting parameter which is zero for unconstrained conditions and nonzero for the constrained coupling coefficient design. In this study case, we find that the best value of α is around 5×10^4, which can achieve an optimal and smoother convergence. The constrain on the value of the coupling constant can be more enforced with the sacrifice of the reflectivity spectrum quality by increasing the values of the weighting parameter β. We choose a value of $\beta=10^{-7}$ for the comparison with the unconstrained situation β=0 in this designed case.

The first synthesized example is a two-channel FBG filter, N=2. The total grating length is L=30mm, the channel spacing is Δ_{CS}= 50GHz and the $\Delta\lambda$ is 0.16nm for each channel corresponding to a bandwidth of full width at half maximum about 0.32nm and bandwidth 0.35nm in -50dB. The simulation results are shown in Fig. 1.The designed reflection spectrum meets excellently with the target spectrum as in Fig. 1(a), with close to 30dB isolation outside the channels and with low dispersion inside the channels (deviation < ± 100 ps/nm in 75% of stopband and the maximum value is about ± 350ps/nm within whole channels). A detailed dispersion profile in one channel of the designed two-channel FBG is shown in Fig. 1(c). The apodization profile of the index modulation for this two-channel FBG filter is shown in Fig. 1(d), with the maximum index modulation only slightly larger than the single-channel one (initial Gaussian apodization). In this work, we only consider the spectral reflectivity amplitude optimization, just to demonstrate the present optimization method. This is why we don't have a flat group delay (dispersionless) response here.

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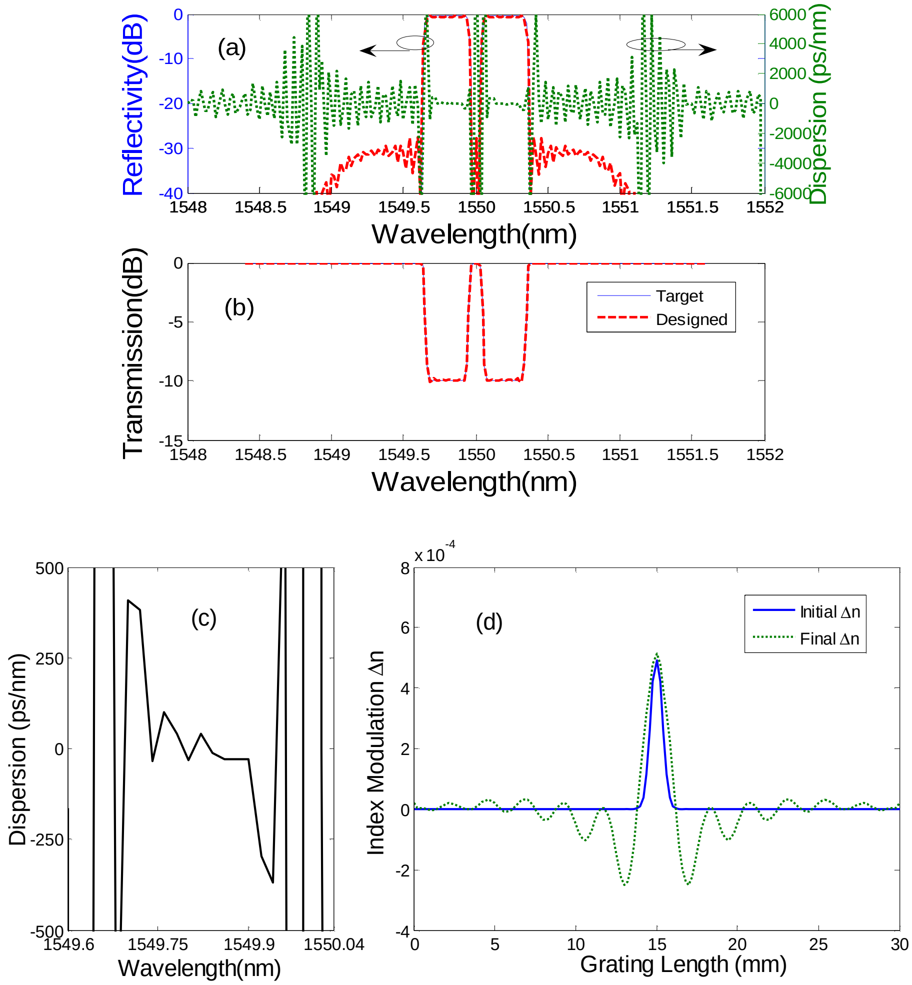

Fig. 1. Two-channel low dispersion FBG filter with channel spacing 50GHz synthesized by the LMO method. (a) Reflection spectrum and dispersion profile, (b) transmission and target spectra, (c) the detailed dispersion profile in one channel, (d) designed apodization profile of the index modulation.

In the following design example, an eight-channel FBG, N=8, with grating length 80mm, channel spacing 25GHz and the $\Delta\lambda$ =0.088nm corresponding to a bandwidth of full width at half maximum 0.16nm and bandwidth 0.2nm in -50dB is synthesized. The simulation results appear in Fig. 2. Again, the designed reflection spectrum meets very well with the target spectrum in Fig. 2(a) with more than 30dB isolation outside the channels. In Fig. 2(c), the dispersion profile in one channel of the 8-channel FBG is shown. The apodization profile of the index modulation for this eight-channel FBG filter is shown in Fig. 2(d), with the maximum index modulation 1.6 times higher than the single-channel one (initial Gaussian apodization). It should be noted that the simulation is finished after hundreds of iterations. The convergence of the LMO method for MCFBG syntheses is efficient and monochromatic. Unlike other phase sampling approaches, no additional Monte-Carlo based optimization algorithm is used here. The typical evolution curves of the calculated average error (total error divided by the number of spectral points) for the cases of N = 2, 4, 8 channel numbers are shown in Fig. 3. The reason why the initial error increases with the channel number is simply because we use the same Gaussian apodization function as the initial guess for all the design cases. When the channel number is increased, the initial error is increased due to the larger mismatch. However, the important thing here is that the convergence behavior (or trend) for different channel numbers is basically the same as can be seen in Fig. 3, despite the different magnitude of the initial errors. That is, the convergence quality actually does not degrade due to the increase of the channel numbers. This proves the suitability of the present method for designing complicated multichannel FBG filters.

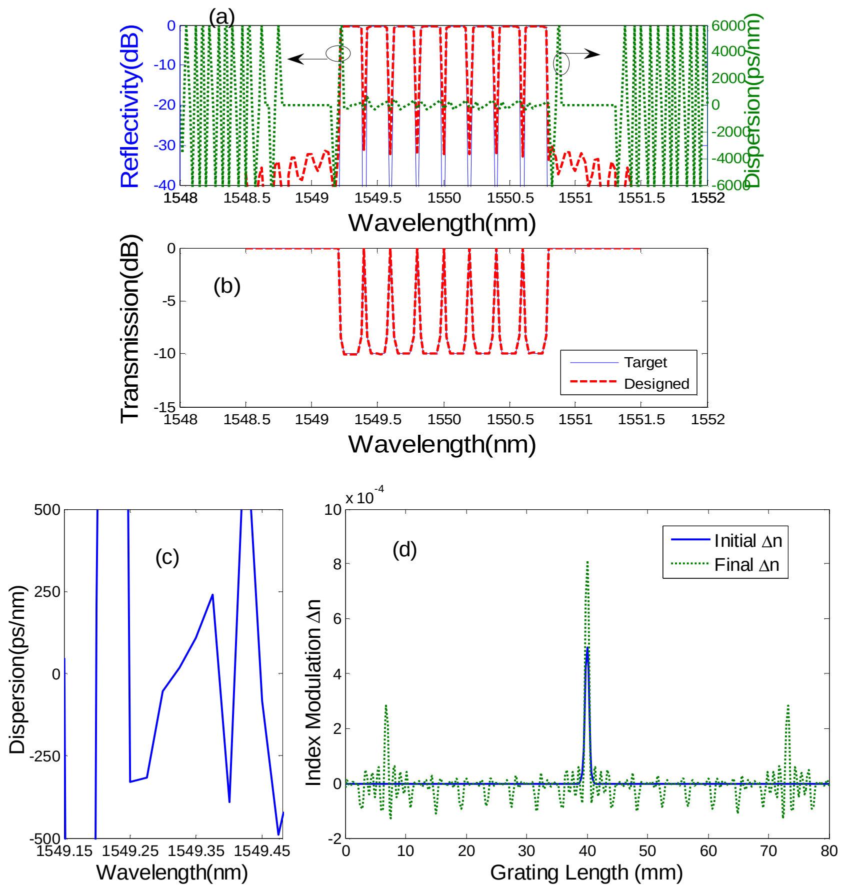

Fig. 2. Eight-channel low dispersion FBG filter with channel spacing 25GHz synthesized by the LMO method. (a) Reflection spectrum and dispersion profile, (b) transmission and target spectra, (c) the detailed dispersion profile in one channel, (d) designed apodization profile of the index modulation.

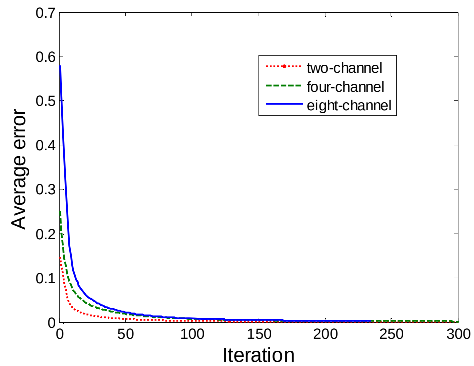

Fig. 3. Typical evolution curves of the average error for the designed MCFBGs in the LMO method.

In the above MCFBG syntheses, the weighting parameter β is zero for the unconstrained design. In this case, the maximum index modulation approximately grows proportional to the square root of the channel numbers, the same rate as a phase sampling approach. To further decrease the maximum value of the index modulation, $\beta = 1\times10^{-7}$ is used to control the maximum index modulation in the apodization profile. In Fig. 4, it can be seen that the maximum index modulation of a MCFBG could be significantly decreased to the same magnitude as the single-channel FBG case by slightly sacrificing the channel reflectivity.

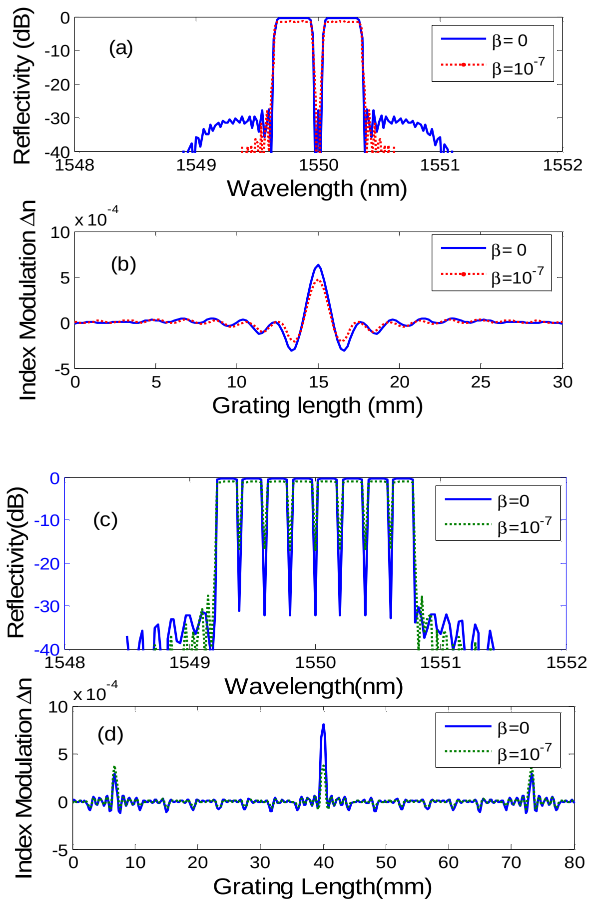

Fig. 4. Reflection spectra and the apodization index profiles for (a), (b) two-channel FBG and (c), (d) eight-channel FBG with the weighting parameter $\beta = 0$ and 1×10^{-7} for unconstrained and constrained coupling coefficient designs, respectively.

4. Conclusion

In conclusion, a novel MCFBG synthesis method based on the Lagrange multiplier optimization (LMO) is presented. Based on the simulation results, it has been found that single stage MCFBGs with arbitrary channel spacing and bandwidth of low dispersion for DWDM applications can be obtained by using the proposed design method. In addition, the maximum amplitude of the index profile of the designed MCFBGs can be reduced by choosing a properly constrained parameter in the LMO algorithm in order to obtain a better index modulation profile for practical photosensitivity fibers. Our future investigation along this line will include the actual fabrication of the designed FBG filters, the dispersion (or other parameters) optimization, and the theoretical extension of the $\kappa(z)$ function to be complex for more design freedom and possible better performance. Finally, it is believed that the proposed method is an attractive and effective way for optimally designing complicated fiber grating devices for practical applications and can be further developed to construct a powerful toolbox for practical design of other optical devices.

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